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Unified dark energy and dark matter from a scalar field different from quintessence

Changjun Gao,¹ Martin Kunz,^{2,3} Andrew R. Liddle,² and David Parkinson²

¹*The National Astronomical Observatories, Chinese Academy of Sciences, Beijing, 100012, China*

²*Astronomy Centre, University of Sussex, Brighton BN1 9QH, United Kingdom*

³*Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH-1211 Genève 4, Switzerland*

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We explore unification of dark matter and dark energy in a theory containing a scalar field of non-Lagrangian type, obtained by direct insertion of a kinetic term into the energy-momentum tensor. This scalar is different from quintessence, having an equation of state between -1 and 0 and a zero sound speed in its rest frame. We solve the equations of motion for an exponential potential via a rewriting as an autonomous system, and demonstrate the observational viability of the scenario, for sufficiently small exponential potential parameter λ , by comparison to a compilation of kinematical cosmological data.

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I. INTRODUCTION

Dark energy remains a fundamental mystery, both in terms of its unexpectedly low but nonzero value and because of the apparent coincidence of its present density being approximately that of other components. Attempts to address the coincidence problem have been of two types. One is to invoke the anthropic principle, perhaps at its most persuasive when coupled with the concept of the string landscape [1]. The second is to permit the dark energy to be a dynamical entity, and hope to exploit solutions of scaling or tracking type to remove dependence on initial conditions (for a review see Ref. [2]). It is, however, fair to say that no compelling scenario of the second type has been found that is compatible with the tight present observational constraints on the equation of state parameter w [3].

In this paper we study the consequences of modeling the dark energy using a scalar field that is of non-Lagrangian type. The principle that fundamental physics should derive from a Lagrangian description is a deep-seated part of modern physics, as powerfully argued for instance by Durrer and Maartens [4]. It is a measure of the difficulty of the dark energy problem that there have been several papers that have abandoned this principle, for instance modeling the dark energy as a phenomenological fluid which exhibits a particular scaling with the scale factor [5] or Hubble parameter [6], or even allowing a cosmological constant with an explicit dependence on time [7]. Our proposal too is of this general type; we are closer to traditional quintessence modeling in adopting a scalar field description, but consider a scalar field that does not emerge from a Lagrangian.

Dropping the Lagrangian assumption is a major step, and in taking such a step one wishes to be sure that there is significant payback. Our model offers one such reward—it permits a unified description of dark energy and dark matter as due to the single field we consider. While our proposal is a speculative one, this opportunity is significant enough to merit study.

Our proposal is not of course the first to seek to unify dark energy and dark matter into a single material. Discounting those where the dark energy arises from a constant term in the action, some examples are as follows. Reference [8] proposed a tachyon-type scalar-field Lagrangian, in which the scalar fluid can be broken up into dark matter and dark energy components. K -essence unification of dark matter and dark energy has been studied in Ref. [9]. Staying instead with the canonical Lagrangian, Ref. [10] introduced a complex scalar field with a mixed potential made of quadratic and exponential terms, which then mimic dark matter and dark energy, respectively. Alternative strands with similar goals are study of the generalized Chaplygin gas [11] and of barotropic fluid models [12].

II. MODIFYING THE EINSTEIN EQUATIONS

The usual Einstein equations are given by

$$G_{\mu\nu} = \kappa^2(T_{\mu\nu} + \Lambda g_{\mu\nu}), \quad (1)$$

where $G_{\mu\nu}$, $T_{\mu\nu}$, and $\Lambda g_{\mu\nu}$ are the Einstein tensor, energy-momentum tensor and the cosmological constant term, respectively. $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant and $\kappa^2 = 8\pi G$. Λ is the simplest version of dark energy, being time independent and isotropically and homogeneously distributed in space. It suffers from the coincidence problem, and to address this we wish to allow the cosmological constant to evolve.

A simple idea, as adopted in quintessence models, is to allow Λ to be a function of some scalar field ϕ . One cannot however allow this dependence to be on ϕ alone; the Bianchi identity

$$\nabla^\mu G_{\mu\nu} = 0, \quad (2)$$

and the law of energy-momentum conservation,

$$\nabla^\mu T_{\mu\nu} = 0, \quad (3)$$

would force

$$\nabla^\mu [\Lambda(\phi)g_{\mu\nu}] = 0, \quad (4)$$

requiring $\Lambda(\phi)$ to still be constant.

To remedy this it is necessary to incorporate a dynamical term, depending on $\nabla^\mu \phi$, into the equations. For quintessence this is done by including a canonical kinetic term in the Lagrangian; $\Lambda(\phi)$ then becomes the scalar-field potential and the total dark energy density includes both potential and kinetic terms. Here we propose the simplest possible alternative, which is the direct insertion of a kinetic term into the energy-momentum tensor:

$$G_{\mu\nu} = \kappa^2 [T_{\mu\nu} + \Lambda(\phi)g_{\mu\nu} - \frac{1}{2}\nabla_\mu \phi \nabla_\nu \phi]. \quad (5)$$

Now $\Lambda(\phi)$ is not necessarily a constant. The equation of motion for the scalar field is then given by

$$\nabla^2 \phi - 2 \frac{d\Lambda}{d\phi} + \frac{\nabla^\mu \phi \nabla^\nu \phi \cdot (\nabla_\mu \nabla_\nu \phi)}{\nabla_\alpha \phi \nabla^\alpha \phi} = 0, \quad (6)$$

This equation follows directly from the Einstein equations (plus the assumption that for the other components $T_{\mu\nu}$ remains separately conserved), but this form is more convenient.

At first glance this scalar field looks very much like a quintessence field. But in fact it is very different from quintessence, and even different from K -essence [13] where the Lagrangian is written as a general function of ϕ and $\nabla^\mu \phi \nabla_\mu \phi$. Indeed, it has no Lagrangian formulation in the framework of K -essence theory.

We can present the proof as follows. In general, the Lagrangian of K -essence is given by an arbitrary function

$$\mathcal{L} = \mathcal{L}(\phi, X), \quad (7)$$

where ϕ is a scalar field and

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi. \quad (8)$$

Since we choose the signature of $(-1, +1, +1, +1)$, we always have $X \geq 0$. Varying this Lagrangian with respect to the metric we obtain the energy-momentum tensor in the form

$$T_{\mu\nu} = \mathcal{L}_{,X} \nabla_\mu \phi \nabla_\nu \phi + \mathcal{L} g_{\mu\nu}, \quad (9)$$

where $\mathcal{L}_{,X}$ denotes partial derivative of the Lagrangian with respect to X . By identifying it with the energy-momentum tensor of the scalar

$$T_{\mu\nu} = -\frac{1}{2}\nabla_\mu \phi \nabla_\nu \phi + \Lambda(\phi)g_{\mu\nu}, \quad (10)$$

we find the corresponding Lagrangian does not exist. This demonstration is limited to Lagrangians of K -essence form, but there is no reason to think that a more general Lagrangian, such as $\mathcal{L}(X, \phi, R_{\mu\nu}\nabla^\mu \phi \nabla^\nu \phi, \dots)$ could lead to our equations while retaining the Einstein form of gravity.

III. DYNAMICAL EQUATIONS

The equations that govern the evolution of the spatially flat Friedmann–Robertson–Walker Universe are

$$\begin{aligned} 3H^2 &= \kappa^2 \left[\frac{1}{2}\dot{\phi}^2 + \Lambda(\phi) + \sum \rho_i \right], \\ 2\dot{H} + 3H^2 &= -\kappa^2 \left[-\Lambda(\phi) + \sum p_i \right], \end{aligned} \quad (11)$$

$$\frac{d\rho_i}{dt} + 3H(\rho_i + p_i) = 0, \quad (12)$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_i and p_i are the energy density and pressure of i th matter component (namely, relativistic matter, baryonic matter, and so on). a is the scale factor and dot denotes derivative with respect to the physical time t . We have set $a_0 = 1$ for the present universe.

From the Einstein equations above we obtain the density and pressure of our scalar

$$\rho_{\text{sca}} = \frac{1}{2}\dot{\phi}^2 + \Lambda(\phi), \quad p_{\text{sca}} = -\Lambda(\phi). \quad (13)$$

These can be contrasted with the equivalents for quintessence with the same potential

$$\rho_{\text{qui}} = \frac{1}{2}\dot{\phi}^2 + \Lambda(\phi), \quad p_{\text{qui}} = \frac{1}{2}\dot{\phi}^2 - \Lambda(\phi). \quad (14)$$

From the expressions of density and pressure, we know quintessence has the equation of state $-1 \leq w_{\text{qui}} \leq 1$ for $\Lambda \geq 0$, while the scalar has $-1 \leq w_{\text{sca}} \leq 0$. From the conservation equation (12) we then know that the density of quintessence scales in the range a^{-6} to a^0 , while for the scalar the range is restricted to a^{-3} and a^0 .¹ This property suggests that the scalar may play the role of both dark matter (scaling approximately as a^{-3}) and dark energy (scaling approximately as a^0).

From the expressions for the density and pressure we can further derive the sound speed in the rest-frame of the scalar fields, and find that they are different as well:

$$\hat{c}_{\text{sca}}^2 = \frac{\partial p / \partial X}{\partial \rho / \partial X} = 0, \quad \hat{c}_{\text{qui}}^2 = \frac{\partial p / \partial X}{\partial \rho / \partial X} = 1, \quad (15)$$

where $X = \dot{\phi}^2/2$. This is the case for any potential $\Lambda(\phi)$. The vanishing of the sound speed allows our scalar field to cluster gravitationally more easily than quintessence. Since this is crucial in order to match the cosmic microwave background (CMB) data, we derive the behavior of the perturbations explicitly later on, confirming this result.

The equation of motion for ϕ can be derived from Eqs. (11) or Eq. (6) as

¹This is the same range accessible to the simplest DBI tachyon model [8,14], but our equation of motion differs from that case.

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + \frac{d\Lambda}{d\phi} = 0. \quad (16)$$

Compared with the equivalent equation for quintessence, with the same potential, there is a significant difference: the friction term in the equation of motion for the scalar is only half that of the quintessence. So with increasing redshift, the densities of the scalar field will increase more slowly than quintessence. In fact, if the potentials are constant or sufficiently flat such that $d\Lambda/d\phi \approx 0$, in a kinetic-dominated regime we will have

$$\rho_{\text{qui}} \propto a^{-6}, \quad (17)$$

for quintessence and

$$\rho_{\text{sca}} \propto a^{-3}, \quad (18)$$

for the scalar. The latter is exactly that of cold dark matter.

IV. THE EVOLUTION OF THE HOMOGENEOUS SCALAR FIELD

In order to compute the evolution of the scalar field and to check whether it is compatible with current data sets, we need to specify a form for the potential $\Lambda(\phi)$. The simplest choice is just a constant potential, $\Lambda = \text{const}$ in Eqs. (11). Then from Eq. (16) we have the density

$$\rho_{\text{sca}} = \Lambda + \frac{\rho_{d0}}{a^3}. \quad (19)$$

where ρ_{d0} is a constant which can be interpreted as the present dark matter density. This is exactly the Λ CDM model, with Λ playing the role of dark energy and $\nabla_\mu \phi \nabla_\nu \phi / 2$ the role of dark matter.

However, taking the potential to be constant is effectively reintroducing a pure cosmological constant (cf. the kinetic K -essence model of Scherrer [9]), and hence does not represent a significant step forward in understanding the nature of dark energy, although our proposal has a novel nature for the dark matter. We therefore choose a more general form of the potential,

$$\Lambda(\phi) = V_0 e^{-\kappa \lambda \phi} \quad (20)$$

which we will use throughout the remainder of the paper. Here V_0 and λ are two constants. Without loss of generality, we assume $\lambda > 0$. In the limit $\lambda \rightarrow 0$ we recover the constant potential case and therefore the model is continuously connected with Λ CDM, at least where the background evolution is concerned.

A. Autonomous system of equations

To study the evolution of the field, we set up an autonomous system. The main equations are given by

$$3H^2 = \kappa^2 [\frac{1}{2}\dot{\phi}^2 + \Lambda(\phi) + \rho_r + \rho_b], \quad (21)$$

$$2\dot{H} + 3H^2 = -\kappa^2 [-\Lambda(\phi) + \frac{1}{3}\rho_r], \quad (22)$$

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} + \frac{d\Lambda}{d\phi} = 0, \quad (23)$$

where ρ_r and ρ_b are the density of radiation and baryonic matter, respectively. They have a constant equation of state equal to $1/3$ and 0 , respectively. Following Ref. [15], we introduce the following dimensionless quantities

$$x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\kappa \sqrt{\Lambda}}{\sqrt{3}H}, \quad (24)$$

$$\sqrt{\Omega_b} \equiv \frac{\kappa \sqrt{\rho_b}}{\sqrt{3}H}, \quad \sqrt{\Omega_r} \equiv \frac{\kappa \sqrt{\rho_r}}{\sqrt{3}H}.$$

Here x^2 and y^2 represent the density parameters of the kinetic and potential terms, respectively. We expect interesting cases to have the scalar field rolling down the slope of the potential, so since we have assumed $\lambda > 0$, we should have $x > 0$. Then the above equations can be written in the following autonomous form

$$\frac{dx}{dN} = -\frac{3}{2}x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x \left[1 - y^2 + \frac{1}{3}(1 - x^2 - y^2 - \Omega_b) \right], \quad (25)$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y \left[1 - y^2 + \frac{1}{3}(1 - x^2 - y^2 - \Omega_b) \right], \quad (26)$$

$$\frac{d\Omega_b}{dN} = -3\Omega_b \left[y^2 - \frac{1}{3}(1 - x^2 - y^2 - \Omega_b) \right], \quad (27)$$

together with a constraint equation

$$x^2 + y^2 + \Omega_b + \Omega_r = 1. \quad (28)$$

Here $N \equiv \ln a$. The equation of state w_ϕ and the fraction of the energy density Ω_ϕ for the scalar field are

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = -\frac{y^2}{x^2 + y^2}, \quad (29)$$

$$\Omega_\phi \equiv \frac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2. \quad (30)$$

B. Observational requirements

What constraints does a model seeking to explain both dark matter and dark energy have to satisfy? Normally, observational constraints are imposed under the assumption of separate dark matter and dark energy components, but due to the dark degeneracy, first described by Hu and Eisenstein [16] and then further explored in Refs. [17–19], gravitational probes alone are unable to give a unique decomposition and can only impose constraints on the total dark sector. In Ref. [20] we recently derived the constraints

TABLE I. The properties of the critical points for the exponential potential given by $\Lambda = V_0 e^{-\kappa\lambda\phi}$.

Name	x	y	$\sqrt{\Omega_b}$	Existence	Stability	Ω_ϕ	w_ϕ
(a)	0	0	0	All λ	Unstable node	0	-
(b)	$\frac{x}{\sqrt{6}}$	0	$\sqrt{1-x^2}$	All λ	Saddle line segment for $0 \leq x \leq 1$	x^2	0
(c)	$\frac{\sqrt{6}}{3}\lambda$	$\sqrt{1-\frac{2}{3}\lambda^2}$	0	$\lambda^2 \leq \frac{3}{2}$	Stable node for $\lambda < \frac{\sqrt{6}}{2}$	1	$-1 + \frac{2}{3}\lambda^2$

on a combined dark sector fluid from current kinematical observations in a model-independent way. These showed that the total dark sector equation of state must start at or near the cold dark matter value $w = 0$, and then evolve to become negative by the present following a particular profile. The standard cosmological model, e.g. as in Komatsu *et al.* [3], predicts a present total dark sector equation of state of about -0.78 (the weighted mean of the dark matter and cosmological constant contributions), but in fact this value is only weakly constrained [20,21]. The behavior is more tightly constrained at higher redshifts, where the actual observational data lies. In addition, a successful unified dark sector model must reproduce the present dark sector density $\Omega_{\text{dark}} = 0.96$. Our aim will be to test whether our model can achieve this.

C. Fixed points and phase portraits

In Table I, we present the properties of the three fixed points for the exponential potential. The point (a) corresponds to the radiation-dominated epoch and this point is unstable. The line segment (b) corresponds to a scalar plus baryon-dominated epoch and it is a saddle line segment. In this epoch, the scalar field behaves as dust matter which has the equation of state $w = 0$. The point (c) corresponds to a scalar-dominated epoch. Point (c) is stable and thus an attractor. In this epoch, the scalar has an equation of state $w < -1/3$ if $\lambda < 1$, and so the Universe accelerates in this epoch.

Viable scenarios start at high redshift near the unstable radiation fixed point (a). This is necessary since the distance from the origin corresponds to the relative energy density in the scalar field. As that energy density, like the one in matter, decreases slower than the radiation energy density, we need to start close to $x = y = 0$, analogous to the “thawing” regime of quintessence. In order to follow the usual evolution of the Universe, the field should then move to the effective matter-dominated saddle line. This happens if we start with $y \ll 1$ as $dy/dN \propto y \approx 0$ in this limit. The trajectories then move across, staying close to the $y = 0$ line, before turning up. Figure 1 shows phase portraits for $\lambda = 0.01$ with various initial conditions. The trajectories are all confined inside the circle given by $x^2 + y^2 = 1$ due to the constraint equation Eq. (28).

The present epoch can be identified through the requirement that $\Omega_{\phi 0} = x^2 + y^2 \simeq 0.96$, which corresponds to a circle just inside the limiting circle $x^2 + y^2 = 1$. Since all trajectories evolve towards the single attractor (c) which

lies on $x^2 + y^2 = 1$, all viable trajectories will cross that line eventually. In addition, the total equation of state parameter of the scalar should be of the order of $w_0 \simeq -0.78$ today. That condition can be graphically represented by a straight radial line with an angle $B \simeq \arcsin(\sqrt{0.78})$ with respect to the x -axis since $w_\phi = -\sin^2(B)$. The good models then need to cross the circle of today's Ω_ϕ at the intersection with this line. We will examine the observational constraints in more detail in the following subsection.

In Fig. 2, we plot the evolution of density fractions for radiation, baryon matter, kinetic term and potential term, for best-fit model parameters we determine below. This

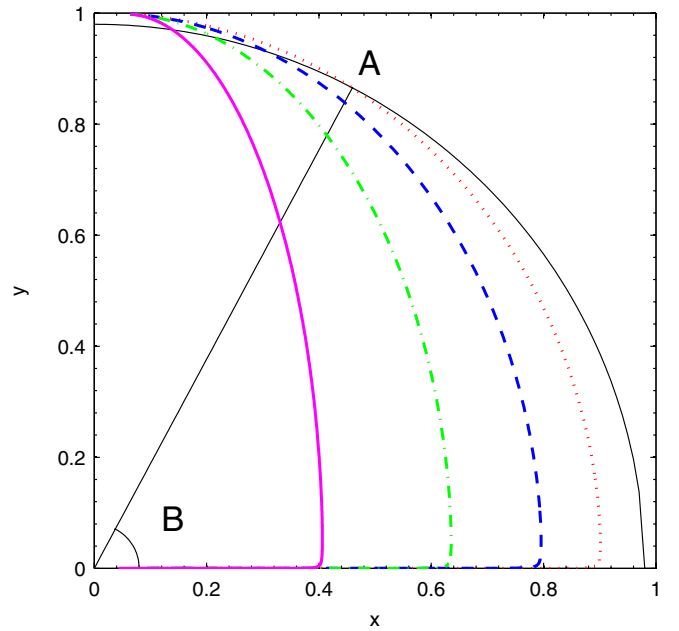


FIG. 1 (color online). The phase plane for the best-fit λ found below, approximately 0.1. The point (0, 0) corresponds to the radiation-dominated epoch. The point (0, 0) is unstable and the point (0.0082, 0.9999) is stable and thus an attractor. The line segment $(x, 0)$ is a saddle line. The initial conditions best matching observations lead to the red/dotted trajectory, while the other trajectories have different initial conditions. The outer thin solid line corresponds to $x^2 + y^2 = 1 - \Omega_{b0} - \Omega_{r0}$, giving the correct present dark sector energy density, while the angle B gives the required equation of state $w_\phi = -\sin^2 B \simeq -0.78$. So the present-day Universe must lie in the vicinity of the point A which is the intersection between the two thin solid lines, which the dotted line indeed passes through.

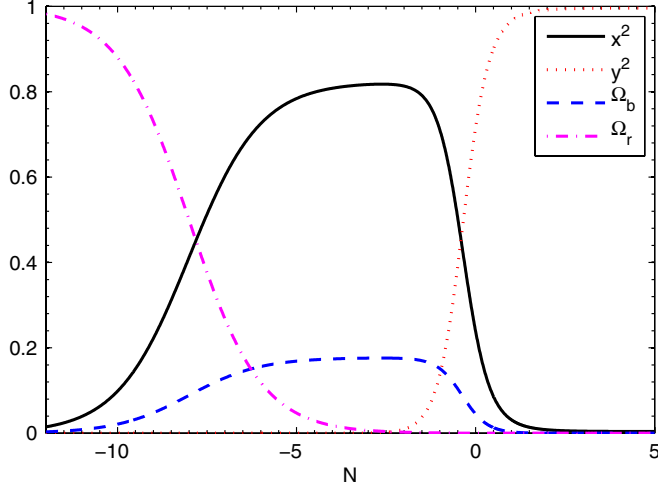


FIG. 2 (color online). The evolution of density fractions for radiation (magenta/dot-dashed line), baryons (blue/dashed line), kinetic term x^2 (black/solid line), and potential term y^2 (red/dotted line), for the best-fit model found in subsection IV D.

shows that the scalar field can mimic the cold dark matter and dark energy very well.

D. Constraints from current data

We now impose detailed observational constraints on our model to establish its viability. We follow a method very similar to that outlined in Ref. [20] to constrain the model, applying a Markov Chain Monte Carlo (MCMC) approach to compute the parameter posterior probabilities. We assume a flat universe, and fix the radiation density today from the CMB temperature. The evolution of the scalar field is defined in terms of the potential parameter λ and the value of the equation of state today w_0 , and then integrated backwards to find its evolution at earlier time. We assume a uniform prior on the equation of state parameter of $-1 < w_0 < 0$ and a log prior on λ such that $-4 < \log_{10}(\lambda) < 1$. We also include the baryon density $\Omega_b h^2$ and the Hubble parameter today H_0 as free parameters, and marginalize over them.

We use a fairly typical compilation of kinematical data. Standard candle data comes from supernova type Ia luminosity distances, for which we use the cut Union supernova sample [22] (with systematic errors included), and standard ruler data comes from the angular positions of the CMB [23] and baryon acoustic oscillation peaks [24]. Note that Ref. [23] gives constraints on the scaled distance to recombination R and the angular scale of the sound horizon l_a . These are defined to be

$$R \equiv \sqrt{\Omega_m H_0^2} r(z_{\text{CMB}}), \quad l_a \equiv \frac{\pi r(z_{\text{CMB}})}{r_s(z_{\text{CMB}})}. \quad (31)$$

Since R is scaled by the physical matter density, and so makes assumptions about the separability of the dark matter and dark energy, we ignore it in this work. We use only

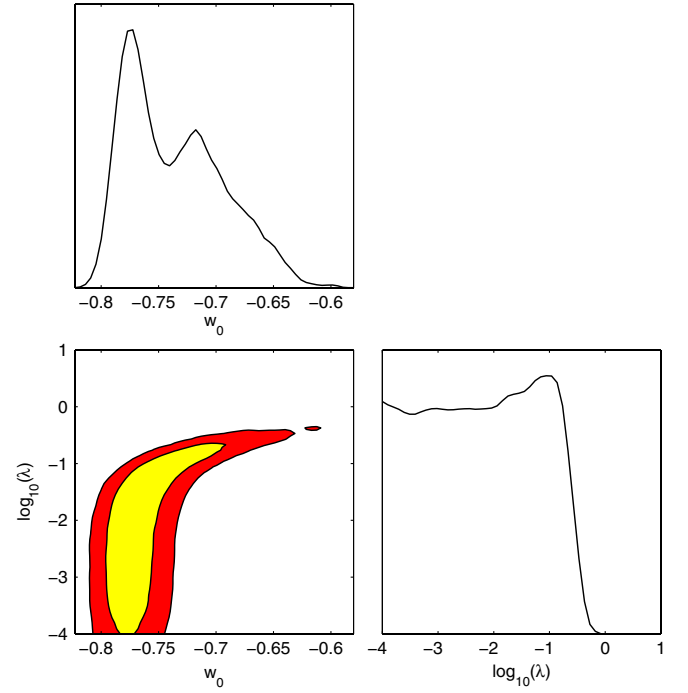


FIG. 3 (color online). The 1-d and 2-d probability distribution for the equation of state today, w_0 , and the potential parameter, $\log_{10}(\lambda)$.

the constraints on l_a , as well as those on $\Omega_b h^2$ and the correlations between the two. We also include the SHOES [25] measurement of the Hubble parameter today, $H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

We find that the model is a good fit to the data. The best-fit parameters have a $\chi^2 = 312.1$, which is almost equiva-

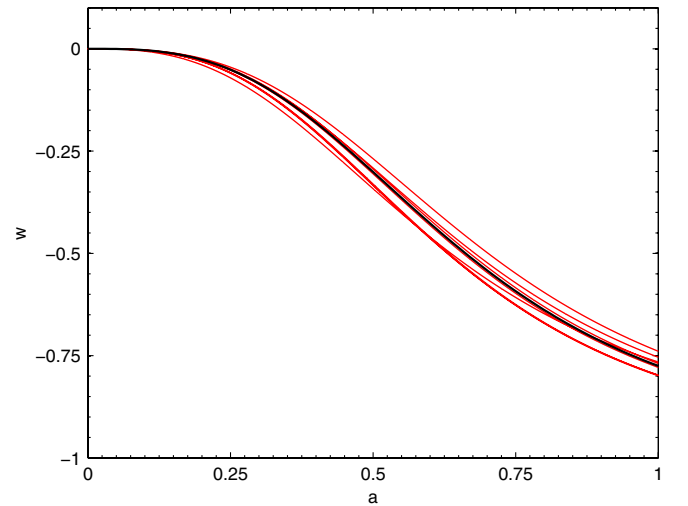


FIG. 4 (color online). The evolution of the equation of state for the scalar, shown for 8 models drawn from the Markov chain. It behaves as the cold dark matter at higher redshifts and dark energy for the lower redshifts. For comparison the evolution of the total w in the Λ CDM model is shown as the thicker black line, which runs more or less centrally through the set.

lent to the best fit of the LCDM model, $\chi^2 = 311.9$ (though the scalar-field model has one extra parameter). The equation of state today lies in the range $-0.82 < w_0 < -0.57$ at 95% confidence. The 95% upper limit on the potential parameter is $\lambda < 0.20$. The probability distributions for these two parameters are shown in Fig. 3 and some sample $w(a)$ curves in Fig. 4.

V. STRUCTURE FORMATION

From the analysis in Ref. [19] we know that the model will fit the CMB data if the rest-frame sound speed is indeed zero. Because this is such an important condition on the model, we here derive the sound speed directly from the perturbation equations. We will discuss the behavior of the perturbations further in the appendix. For this purpose, we work in Newtonian gauge. In the absence of anisotropic stress, and for scalar perturbations, the perturbed Friedmann–Robertson–Walker metric can be written in the form

$$ds^2 = -(1 + 2\Phi)dt^2 + a(t)^2(1 - 2\Phi)dx_i dx^i, \quad (32)$$

where Φ is the gauge-invariant Newtonian potential. The potential characterizes the metric perturbations.

For the dark matter and dark energy dominated Universe, we can safely neglect the effect of radiation. We assume baryonic matter as a perfect fluid which has the energy-momentum tensor

$$T_{\mu\nu} = (\rho_b + p_b)u_\mu u_\nu + p_b g_{\mu\nu}, \quad (33)$$

where u_μ is the four-velocity of the fluid. Perturbations in the energy density ρ_b , pressure p_b and four-velocity u_μ can be written as

$$\begin{aligned} \rho_b(t, \vec{x}) &= \rho_{0b} + \delta\rho_b(t, \vec{x}), \\ p_b(t, \vec{x}) &= p_{0b} + \delta p_b(t, \vec{x}), \\ u_\mu(t, \vec{x}) &= {}^{(0)}u_\mu + \delta u_\mu(t, \vec{x}), \end{aligned} \quad (34)$$

where ${}^{(0)}u_\mu = (-1, 0, 0, 0)$ and $\rho_{0b}(t)$, $p_{0b}(t)$ are the homogeneous and isotropic energy density and pressure. So we obtain

$$\begin{aligned} \delta T_0^0 &= \delta\rho_b(t, \vec{x}), & \delta T_0^i &= (\rho_{0b} + p_{0b})\delta u^i(t, \vec{x}), \\ \delta T_i^j &= -\delta p_b(t, \vec{x})\delta_i^j. \end{aligned} \quad (35)$$

For the scalar field, we define the perturbation as

$$\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x}). \quad (36)$$

From the energy-momentum tensor

$$T_{\mu\nu} = -\frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi + \Lambda(\phi)g_{\mu\nu}, \quad (37)$$

we get the perturbed counterpart

$$\begin{aligned} \delta T_0^0 &= \delta\rho_\phi = \dot{\phi}_0\delta\phi - \Phi\dot{\phi}_0^2 + \frac{d\Lambda}{d\phi}\delta\phi, \\ ik\delta T_0^i &= ik(\rho_{\phi 0} + p_{\phi 0})\delta u_\phi^i(t, \vec{x}) = \frac{k^2}{2a}\dot{\phi}_0\delta\phi \equiv \rho_{\phi 0}V, \\ \delta T_i^j &= -\delta p_\phi(t, \vec{x})\delta_i^j = \frac{d\Lambda}{d\phi}\delta\phi\delta_i^j. \end{aligned} \quad (38)$$

Since we are working in linear perturbation theory, it is convenient to transform the equations from real space to Fourier space since each Fourier mode evolves independently. We will also suppress the 0-subscripts for the homogeneous and isotropic quantities from now on.

It is well known that both the adiabatic sound speed and the rest frame sound speed (the sound speed for the fluid in its rest frame) play a very important role in the discussion of structure formation theory. Here we work out the two quantities explicitly. The adiabatic sound speed squared is defined through [26]

$$c_a^2 \equiv \frac{\dot{p}_\phi}{\dot{\rho}_\phi} = \frac{2}{3H\dot{\phi}} \frac{d\Lambda}{d\phi}. \quad (39)$$

The rest frame sound speed squared \hat{c}_s^2 of the scalar is related to the pressure perturbation in the Newtonian gauge through

$$\delta p_\phi = \hat{c}_s^2 \delta\rho_\phi + \frac{3aH}{k^2}(\hat{c}_s^2 - c_a^2)\rho_\phi V. \quad (40)$$

Expressing this equation with perturbation quantities, following the procedure in Ref. [26], we find

$$\begin{aligned} -\frac{d\Lambda}{d\phi}\delta\phi &= \hat{c}_s^2\left(\dot{\phi}\delta\phi - \dot{\phi}^2\Phi + \frac{d\Lambda}{d\phi}\delta\phi + 3H\dot{\phi}\delta\phi\right) \\ &\quad - \frac{d\Lambda}{d\phi}\delta\phi. \end{aligned} \quad (41)$$

Therefore, we can conclude immediately that

$$\hat{c}_s^2 = 0. \quad (42)$$

This is consistent with the discussion in Eq. (15). The vanishing sound speed in the rest frame of the scalar allows it to play the role of cold dark matter.

We have the perturbation for the pressure as follows

$$\delta p_\phi = -c_a^2 \frac{3aH}{k^2} \bar{\rho} V. \quad (43)$$

Both CDM and a cosmological constant have $\delta p = 0$. How large is the contribution to δp which arises from the gauge transformation to Newtonian gauge due to $c_a^2 \neq 0$? The adiabatic sound speed squared can be written as

$$c_a^2 = w_\phi - \frac{\dot{w}_\phi}{3H(1 + w_\phi)}. \quad (44)$$

We note that the adiabatic sound speed is determined by the homogeneous quantities. To have a picture of the adiabatic sound speed, we should resort to the background

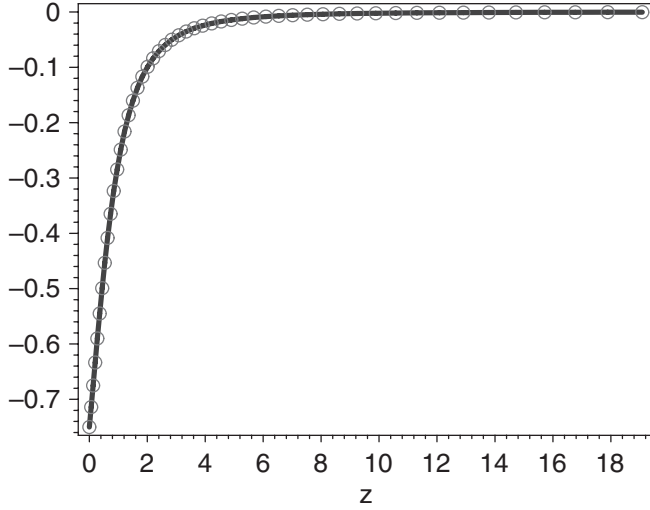


FIG. 5. The adiabatic sound speed squared c_a^2 (solid line) and the equation of state (circled line) for the scalar, showing $c_a^2 \approx w_\phi$. At redshifts greater than 6, they are both vanishing.

equations, given by

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_b + \frac{1}{2} \dot{\phi}^2 \right), \quad (45)$$

$$\ddot{\phi} + \frac{3}{2} H \dot{\phi} + \frac{d\Lambda}{d\phi} = 0. \quad (46)$$

Using our best-fit parameters, in Fig. 5, we plot the equation of state w_ϕ and the adiabatic sound speed squared c_a^2 for the scalar. It shows that $c_a^2 \approx w_\phi$. At the redshifts greater than 6, both the equation of state w_ϕ and the adiabatic sound speed c_a are nearly zero and unimportant for structure formation. This point is essential for the scalar to play the role of cold dark matter. In conclusion, we have $c_a \approx 0$, $w_\phi \approx 0$ at redshifts greater than about 6. Thus we also have $\delta p \approx 0$ at redshifts greater than 6, from Eqs. (43) and (44).

The appendix further explores the properties of the structure formation equations.

VI. CONCLUSION

We have investigated in a cosmological context the behavior of a scalar field with nonstandard kinetic term in the Einstein equations. So far we lack a Lagrangian description for this scalar, and we have shown that it is not possible to build one in the framework of K -essence fields. We have not been able to exclude that a more general Lagrangian, such as $\mathcal{L}(X, \phi, R_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi, \dots)$ could mimic our equations while retaining the Einstein form of gravity, or that the Lagrangian looked for could exist in the framework of Kaluza–Klein theories, but equally we have no reason to think it will.

On the plus side, we find that this scalar has some interesting properties. In the first place, it has an equation of state between $w = -1$ and $w = 0$. This is different from the quintessence field which has the equation of state between $w = -1$ and $w = +1$. Hence the scalar field can behave as pressureless matter in the matter- or radiation-dominated epochs, later evolving to take on dark energy properties as well. A degree of fine-tuning is needed in the initial conditions in order to ensure that the scalar field only dominates in the latter stages of the evolution, which is of the same form as that invoked in thawing quintessence models.

Secondly, the rest frame sound speed of the scalar is zero. This is different from quintessence for which the sound speed is equal to the speed of light. Although a DBI scalar field has the same range of equation of state as our scalar, its rest frame sound speed is nonvanishing. As is known, a vanishing sound speed is sufficient for a scalar field to play the role of cold dark matter in the process of structure formation, and the sound speed should not be too big if gravitational collapse is to match current observations [19]. Thanks to the vanishing sound speed of our scalar, we find that it behaves exactly as cold dark matter in the process of structure formation.

To conclude, we have invoked a scenario in which a non-Lagrangian scalar field is able to play the roles of both dark matter and dark energy. With sufficient tuning of initial conditions, we have shown that a satisfactory evolution can be arranged, with present data constraining an exponential potential to have an exponent of 0.2 or less, though the model does not significantly improve on the fit of the Λ CDM model. Our model would be falsified in the event of direct detection of conventional dark matter particles. Whether it can be motivated in terms of fundamental theories of physics, perhaps in an effective theory formulation, remains to be seen.

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APPENDIX: EVOLUTION OF THE PERTURBATIONS

In order to make numerical calculations, we should rewrite the equations in the dimensionless form. To this

end, we use the variable N

$$\frac{\partial}{\partial t} = H \frac{\partial}{\partial N}, \quad \frac{\partial^2}{\partial t^2} = H^2 \frac{\partial^2}{\partial N^2} + HH' \frac{\partial}{\partial N}. \quad (\text{A1})$$

Here prime denotes the derivative with respect to N . So the background equations become

$$HH' = -\frac{\kappa^2}{2} \left(\rho_b + \frac{1}{2} H^2 \phi'^2 \right), \quad (\text{A2})$$

$$H^2 \phi'' + \frac{1}{2} (3H^2 + 2HH') \phi' + \frac{d\Lambda}{d\phi} = 0. \quad (\text{A3})$$

Define

$$h = \frac{H}{H_0}, \quad \Omega_\lambda = \frac{\kappa^2 V_0}{3H_0^2}, \quad \Omega_{b0} = \frac{\kappa^2 \rho_{b0}}{3H_0^2}, \quad (\text{A4})$$

where ρ_{b0} , H_0 are the energy density of baryon matter and the Hubble constant in the present-day Universe. Using these new variables, we can rewrite the main equations in the dimensionless form

$$hh' = -\frac{3}{2} \Omega_{b0} e^{-3N} - \frac{\kappa^2}{4} h^2 \phi'^2, \quad (\text{A5})$$

$$h^2 \phi'' + \frac{h}{2} (3h + 2h') \phi' - 3 \frac{\lambda}{\kappa} \Omega_\lambda e^{-\kappa \lambda \phi} = 0. \quad (\text{A6})$$

We can absorb κ into ϕ . Then the main equations are simplified to be

$$hh' = -\frac{3}{2} \Omega_{b0} e^{-3N} - \frac{1}{4} h^2 \phi'^2, \quad (\text{A7})$$

$$h^2 \phi'' + \frac{h}{2} (3h + 2h') \phi' - 3\lambda \Omega_\lambda e^{-\lambda \phi} = 0. \quad (\text{A8})$$

The perturbed Einstein equations are [27]

$$3H^2 \Phi + 3H\dot{\Phi} + \frac{k^2}{a^2} \Phi = -\frac{\kappa^2}{2} (\rho_b \delta_b + \rho_\phi \delta_\phi), \quad (\text{A9})$$

$$k^2 H \Phi + k^2 \dot{\Phi} = \frac{\kappa^2}{2} a [(\rho_b + p_b) \theta_b + (\rho_\phi + p_\phi) \theta_\phi], \quad (\text{A10})$$

$$\ddot{\Phi} + 4H\dot{\Phi} + (2\dot{H} + 3H^2) \Phi = \frac{\kappa^2}{2} \left(-\frac{d\Lambda}{d\phi} \delta\phi \right), \quad (\text{A11})$$

where $\delta_{b,\phi} \equiv \delta\rho_{b,\phi}/\rho_{b,\phi}$ is the density contrast for baryon matter and scalar, respectively, and $\theta_{b,\phi} \equiv i\vec{k} \cdot \vec{v}_{b,\phi}$ represents the divergence of velocity for baryon matter and scalar, respectively. We note that δ_ϕ is different from the perturbation of the scalar, $\delta\phi$.

On the other hand, the energy conservation equation (which includes the continuity and Euler equations) holds for the baryon matter and the scalar field, respectively. So we obtain [27]

$$\dot{\delta}_b = -\frac{\theta_b}{a} + 3\dot{\Phi}, \quad (\text{A12})$$

$$\dot{\theta}_b = -H\theta_b + \frac{k^2 \Phi}{a}, \quad (\text{A13})$$

for baryon matter [27], and

$$\dot{\delta}_\phi = -(1 + w_\phi) \left(\frac{\theta_\phi}{a} - 3\dot{\Phi} \right) - 3H \left(\frac{\delta p_\phi}{\delta \rho_\phi} - w_\phi \right) \delta_\phi, \quad (\text{A14})$$

$$\begin{aligned} \dot{\theta}_\phi = & -H(1 - 3w_\phi) \theta_\phi - \frac{\dot{w}_\phi}{1 + w_\phi} \theta_\phi + \frac{\delta p_\phi / \delta \rho_\phi}{1 + w_\phi} \frac{k^2}{a} \delta_\phi \\ & + \frac{k^2}{a} \Phi, \end{aligned} \quad (\text{A15})$$

for the scalar. Here

$$\frac{\delta p_\phi}{\delta \rho_\phi} = \frac{-\frac{3}{2} H \dot{\phi} \delta\phi [w_\phi - \frac{\dot{w}_\phi}{3H(1+w_\phi)}]}{\dot{\phi} \delta\phi - \dot{\phi}^2 \Phi + \frac{d\Lambda}{d\phi} \delta\phi}, \quad (\text{A16})$$

$$w_\phi = \frac{-\Lambda}{\frac{1}{2} \dot{\phi}^2 + \Lambda}. \quad (\text{A17})$$

The perturbation equation for the scalar is given by

$$\ddot{\delta\phi} + \frac{3}{2} H \dot{\delta\phi} + \frac{k^2}{2a^2} \delta\phi + 2\Phi \frac{d\Lambda}{d\phi} - \frac{5}{2} \dot{\Phi} \dot{\phi} + \frac{d^2 \Lambda}{d\phi^2} \delta\phi = 0. \quad (\text{A18})$$

We find it is convenient to consider the following equations

$$\ddot{\Phi} + 4H\dot{\Phi} + (2\dot{H} + 3H^2) \Phi = \frac{\kappa^2}{2} \left(-\frac{d\Lambda}{d\phi} \delta\phi \right), \quad (\text{A19})$$

$$\ddot{\delta\phi} + \frac{3}{2} H \dot{\delta\phi} + \frac{k^2}{2a^2} \delta\phi + 2\Phi \frac{d\Lambda}{d\phi} - \frac{5}{2} \dot{\Phi} \dot{\phi} + \frac{d^2 \Lambda}{d\phi^2} \delta\phi = 0, \quad (\text{A20})$$

$$\dot{\delta}_\phi = -(1 + w_\phi) \left(\frac{\theta_\phi}{a} - 3\dot{\Phi} \right) - 3H \left(\frac{\delta p_\phi}{\delta \rho_\phi} - w_\phi \right) \delta_\phi, \quad (\text{A21})$$

$$\begin{aligned} \dot{\theta}_\phi = & -H(1 - 3w_\phi) \theta_\phi - \frac{\dot{w}_\phi}{1 + w_\phi} \theta_\phi + \frac{\frac{\delta p_\phi}{\delta \rho_\phi}}{1 + w_\phi} \frac{k^2}{a} \delta_\phi \\ & + \frac{k^2}{a} \Phi, \end{aligned} \quad (\text{A22})$$

Using the definition of

$$\mathfrak{K} = \frac{k}{H_0}, \quad \Theta_\phi = \frac{\theta_\phi}{H_0}, \quad (\text{A23})$$

and the variable N , we can rewrite the above equations in the dimensionless form

$$h^2\Phi'' + (4h^2 + hh')\Phi' + (2hh' + 3h^2)\Phi = \frac{3}{2}\Omega_\lambda\lambda e^{-\lambda\phi}\delta\phi, \quad (\text{A24})$$

$$h^2\delta\phi'' + \frac{1}{2}(3h^2 + 2hh')\delta\phi' + \frac{\mathfrak{R}^2}{2a^2}\delta\phi - \frac{5}{2}h^2\Phi'\phi' - 6\lambda\Omega_\lambda e^{-\lambda\phi}\Phi + 3\Omega_\lambda\lambda^2 e^{-\lambda\phi}\delta\phi = 0, \quad (\text{A25})$$

$$\delta\phi' = -(1 + w_\phi)\left(\frac{\Theta_\phi}{ah} - 3\Phi'\right) - 3\left(\frac{\delta p_\phi}{\delta\rho_\phi} - w_\phi\right)\delta\phi, \quad (\text{A26})$$

$$\Theta_\phi' = -(1 - 3w_\phi)\Theta_\phi - \frac{w_\phi'}{1 + w_\phi}\Theta_\phi + \frac{\frac{\delta p_\phi}{\delta\rho_\phi}}{1 + w_\phi}\frac{\mathfrak{R}^2}{ah}\delta\phi + \frac{\mathfrak{R}^2}{ah}\Phi. \quad (\text{A27})$$

To be consistent with the discussions of the background equations, we have rescaled ϕ by ϕ/κ as was done earlier. Correspondingly, we have here

$$\frac{\delta p_\phi}{\delta\rho_\phi} = \frac{-\frac{3}{2}h^2\phi'\delta\phi[w_\phi - \frac{w_\phi'}{3(1+w_\phi)}]}{h^2\phi'\delta\phi' - h^2\phi'^2\Phi - 3\lambda\Omega_\lambda e^{-\lambda\phi}\delta\phi}, \quad (\text{A28})$$

$$w_\phi = \frac{-3\Omega_\lambda e^{-\lambda\phi}}{\frac{1}{2}h^2\phi'^2 + 3\Omega_\lambda e^{-\lambda\phi}}. \quad (\text{A29})$$

Now we have Φ , $\delta\phi$, $\delta\phi$, and Θ_ϕ , totalling four perturbation variables, and four differential equations, namely, Eqs. (A24)–(A27). Thus the system of equations is closed.

At redshifts greater than 6, we have $w_\phi \simeq 0$, $c_a^2 \simeq 0$, and $\delta p_\phi/\delta\rho_\phi \simeq 0$. So the perturbation equations simplify to

$$h^2\Phi'' + (4h^2 + hh')\Phi' + (2hh' + 3h^2)\Phi = 0, \quad (\text{A30})$$

$$\delta\phi' = -\left(\frac{\Theta_\phi}{ah} - 3\Phi'\right), \quad (\text{A31})$$

$$\Theta_\phi' = -\Theta_\phi + \frac{\mathfrak{R}^2}{ah}\Phi. \quad (\text{A32})$$

These are none other than the perturbation equations for cold dark matter in Λ CDM model. Therefore, the scalar really behaves as cold dark matter in the process of cosmic structure formation.

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